Spectral structure of beating tsunamis observed in Japan

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Abstract

We studied the spectral structure of beating tsunamis. Selecting 10 beating tsunamis from tide gauge records of tsunamis in Japan we decomposed them into the spectra. We extracted the spectral components contributing to the beat under the assumption that a beat consists of two sinusoidal waves and compared them with dominant frequencies of background oscillations, which had been observed around the same tide stations as the records were obtained. The two spectral components are found as the highest ones in dominant frequencies. Since one of them was equal to fundamental or higher modes of the background oscillation in the frequency, we concluded that tsunami excited some mode of the background oscillation and it caused a resonance of the bay or port. Large decays of the spectra found in the frequency range above the exciting frequencies result from a defect of the higher frequency components in the incident tsunami.

Introduction

A typical beating wave is formed from superposition of two sinusoidal waves with the same amplitude and a little bit different frequency. It is equal to an amplitude moderated wave. Tsunami is a long sea wave mainly excited by earthquakes being generated under the sea bottom. It is observed at tide stations. Japanese tide stations are located in a seismic active zone including circum Pacific seismic zone and frequently record tsunamis. We can sometimes find a beating waveform in them. Tide stations are usually located at the port in the coast and receive an effect of a local oscillation proper to the half-enclosed coast. The effect is reflected in the spectra of the background measured around the tide station in the calm sea condition. As the background spectra there are data measured by Abe (2005a,b, 2006, 2009, 2010a,b) in the coast of Japan. Recently Abe (2011a,b) repeated the measurement in the Pacific coast of Japan. We can compare the tsunami spectra with the background spectra observed around the tide stations. It is interesting to study spectral structure of the beating tsunamis.

Data and analysis

One of criterions of the beating is amplitude-modulated wave with a uniform frequency. Ten beating tsunamis were selected from tide gauge records, which had been observed by Japan Meteorological Agency, Hydrographic and Oceanographic Department, Japan Coastal Guard, and Hokkaido, Tohoku and Hokuriku Regional Development Bureaus. Locations of the tide stations and sources of the tsunamis are tabulated in Table 1 and the map of the tide stations is shown in Figure 1. Sea levels of the records were digitized using a digitizer and the tidal level was reduced from the sea level. After that response corrections were conducted for tide stations with intake pipes using a method and recovery times of Satake et al.(1988). For tide stations with no recovery data we assumed an average recovery time of 269 s used by Abe (2003). The response corrected waveforms are shown in Figure 2. We decomposed them into the power spectra using Goertzel method. Hanning window and moving average were applied to obtain a smoothed spectral form.

Power spectra

The power spectral density were calculated in the frequency range of 0.06-2.4 mHz (6.9-278 minutes in

Table 1 Locations of tide station and tsunami. Location of tsunami is represented by the earthquake epicenter.

No	station name	station lat.	station long.	tsunami name	tsunami lat.	tsunami long.
1	Urakawa	42° 10′ N	142° 46′ E	1989 Sanriku	39° 51′ N	143° 03′ E
2	Urakawa	42° 10′ N	142° 46′ E	1994 Sanriku	40° 27′ N	143° 43′ E
3	Kuji	40° 12′ N	141° 48′ E	2007 Peru	13° 21′ N	76° 31′ W
4	Kamaishi	39° 16′ N	141° 53′ E	1994 Hokkaidotoho-oki	43° 37′ N	147° 43′ E
5	Ayukawa	38° 18′ N	141° 30′ E	1993 Mariana	12° 59′ N	144° 48′ E
6	Onahama	36° 56′ N	140° 54′ E	1995 NorthChile	23° 22′ S	70° 19′ W
7	Chichijima	27° 06′ N	142° 12′ E	1996 Irianjaya	0° 57′ S	137° 01′ E
8	Tsuruga	35° 40′ N	136° 04′ E	1964 Niigata	38° 21′ N	139° 11′ E
9	Owase	34° 05′ N	136° 12′ E	1993 Mariana	12° 59′ N	144° 48′ E
10	Tosashimizu	32° 47′ N	132° 58′ E	1996 Irianjaya	0° 57′ S	137° 01′ E

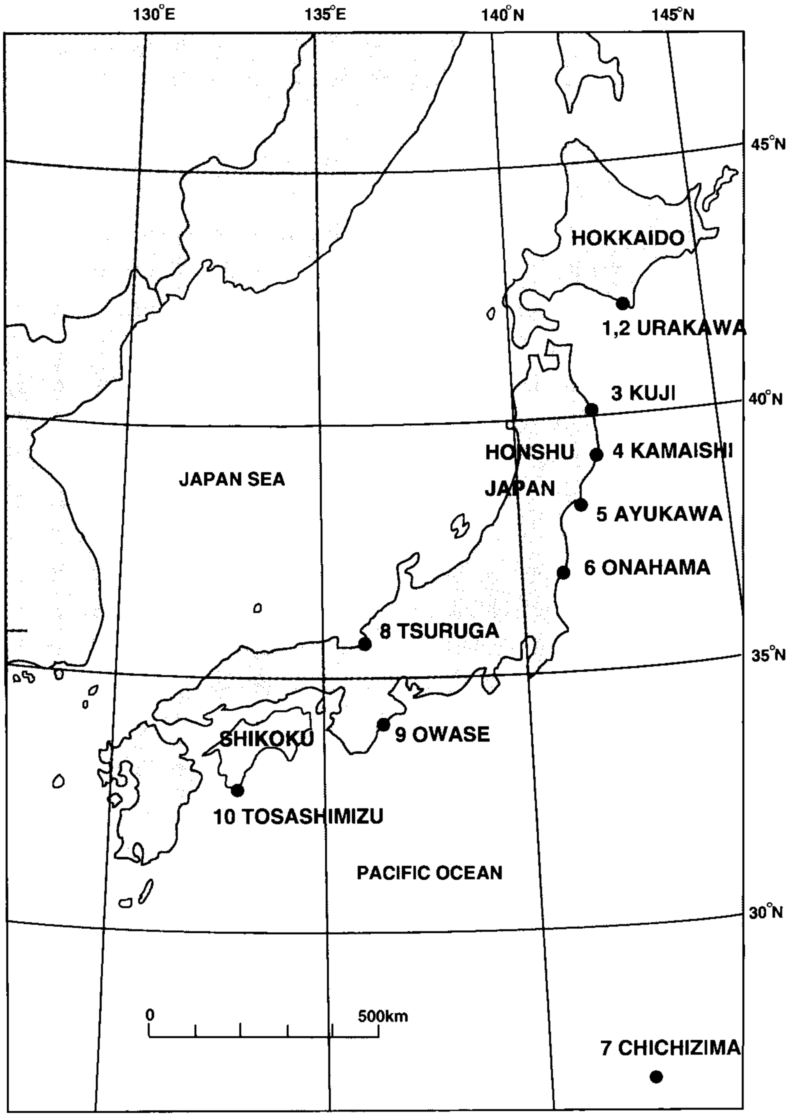

Figure 1 Map of tide stations. Numbers beside the station correspond to ones of beating tsunamis.

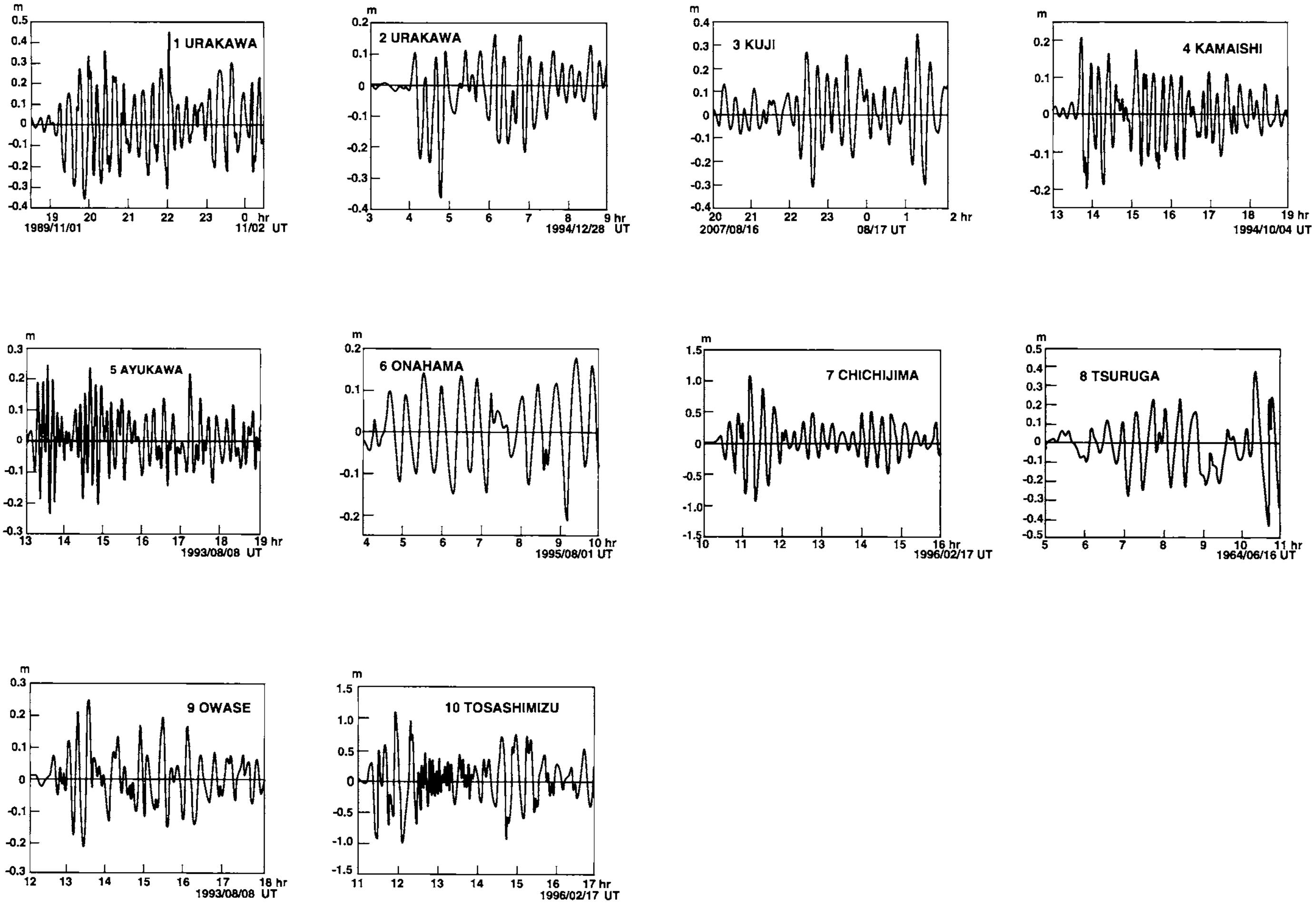

Figure 2 Waveforms of beating tsunamis.

period) and shown in Figure 3. The result shows that most spectra are formed of a band structure. Exceptional cases are No.5 at Ayukawa and No.9 at Owase. They consist of two or three peaks with comparable power amplitude. They have the same source as the 1993 Mariana tsunami but there is a large difference in the excited frequency distribution. Except for these two cases the most dominant frequencies are found in the higher limit of the frequency component. Frequency components contributing to the beat are found in the highest frequencies as described later. Among them No.7 at Chichijima and No.10 at Tosashimizu have a same source as the 1996 Irianjaya tsunami. No.1 and No.2 are obtained at the same tide station of Urakawa but for the different tsunami. In both the cases a kind of similarity structure is observed except for a small displacement of an excited frequency range.

Models of beating tsunamis

1 Model of No3

We approximate the beating tsunami using two frequency components extracted from the power spectra. First case is No.3 at Kuji. The approximated wave is formed from a superposition of two sinusoidal waves as a function of time t having the form of

$$y(t)=a_1 sin(2\pi f_1 t)+a_2 sin(2\pi f_2 t) \quad (1)$$

in which a_1, a_2 are amplitudes and f_1, f_2 are frequencies. We take a_1, : 0.070 m, a_2 : 0.057m ($a_1 > a_2$)and f_1 : 0.0011 Hz (15 min in period), f_2 : 0.0010 Hz (17 min in period) in the model. These values were determined on the obtained spectral amplitude. The wave calculated using the formula (1) and these values is shown in Figure 4 with the observed one. The characteristic property of

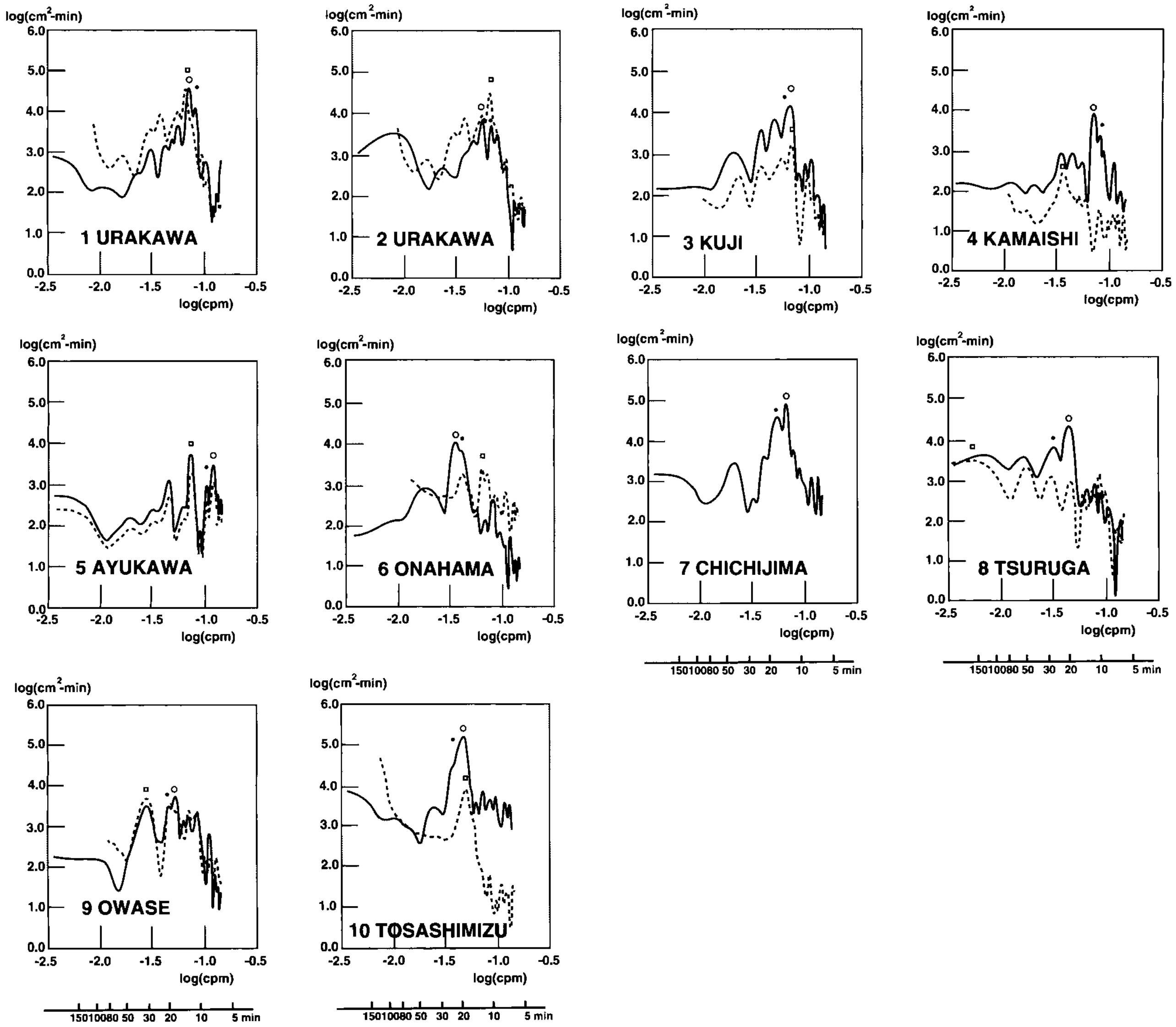

Figure 3 Spectral density of the beating tsunamis (solid lines) and one of seiches after Abe (2009, 2011 a,b) (dotted lines). Open and solid circles represent main spectral components consisting of the beating tsunamis. Open rectangles represent dominant frequencies of the background spectra.

the modulation observed is reproduced in the model.

2 Model of No5

Model of No.5 at Ayukawa is made using the same formula as 1 from a combination of the parameters as a_1, : 0.0281 m, a_2 : 0.0152m ($a_1 > a_2$) and f_1 : 0.00206 Hz (8.1 min in period), f_2 : 0.0174 Hz (9.6 min in period). The superposed wave is shown in Figure 5 with the observed one. The general trend of the modulation is reproduced especially in the early stage of the tsunami. This tsunami changed frequency of the oscillation with the lapse of time and it is supported by the spectra of many peaks. This is an example of beating in a short time.

For other tsunamis we applied the same method to restore the observed beating. Parameters of models as a_1,a_2 and f_1,f_2 were determined using the formula (1) in the same method as 1. The values of f_1, f_2 are inserted in the spectra of Figure 2 with an open circle for f_1 and a solid circle for f_2.

Comparison of the spectra with those of background

From the results obtained by Abe (2009, 2011a,b)

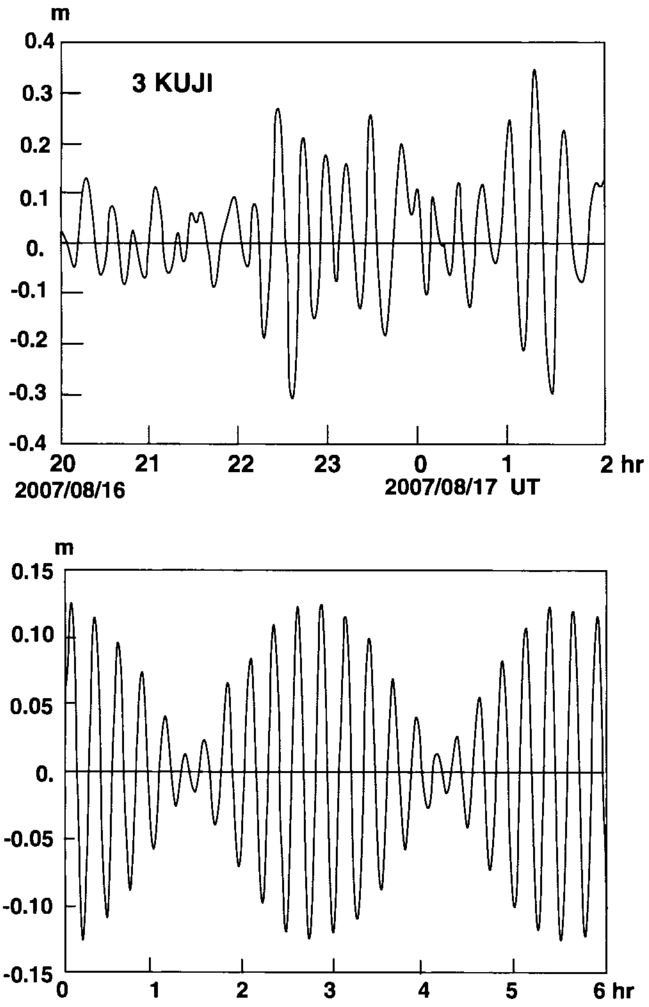

Figure 4 Superposed waveform of two sinusoidal waves (bottom) explaining the beating tsunami (No.3) observed at Kuji tide station (top). The latter one is the same as one of Figure 3. Frequencies and amplitudes of the sinusoidal waves are explained in the text.Map

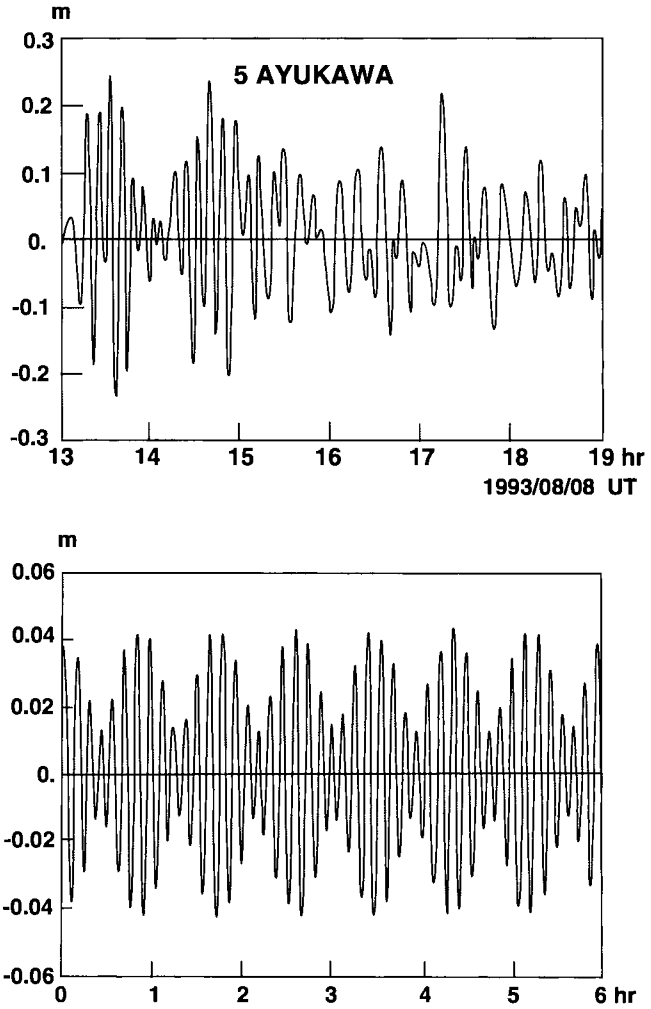

Figure 5 Superposed waveform of two sinusoidal waves (bottom) explaining the beating tsunami (No.5) observed at Ayukawa tide station (top). Same comment as Figure 4

we can find the background spectra observed around tide stations referred here and use them. We reproduced them in Figure 3 of the power spectra and notice the most dominant frequency f_0 shown with open rectangles in the figure. Taking a ratio of beat frequency f_1 to the most dominant frequency of background f_0 we illustrate the relation with each tsunami as shown in Figure 6. The result shows that the ratios consist of some integers except for No. 6 of 0.5 and No. 7 with no data. They are four cases of 1, two cases of 2, a pair of one case of 3 and 7. This fact shows that the beating frequency is related to most dominant frequency of the background. The relation of odd number of the ratio is explained as the fundamental mode and the higher mode of the most dominant frequency. The relation of even number, 2, is explained by a displacement of node position from mouth to inside of a bay. This is the cases of No.4 at Kamaishi and No.9 at Owase We can state that a reason to the displacement is an oblique incidence of the tsunami, which is different from the case of the background. The ratio of 0.5, that is a case of No.6 at Onahama, is the opposite relation of the former case. This tide station almost faces to the open sea and the dominant frequency depends on the incident angle. It is possibly reflect-

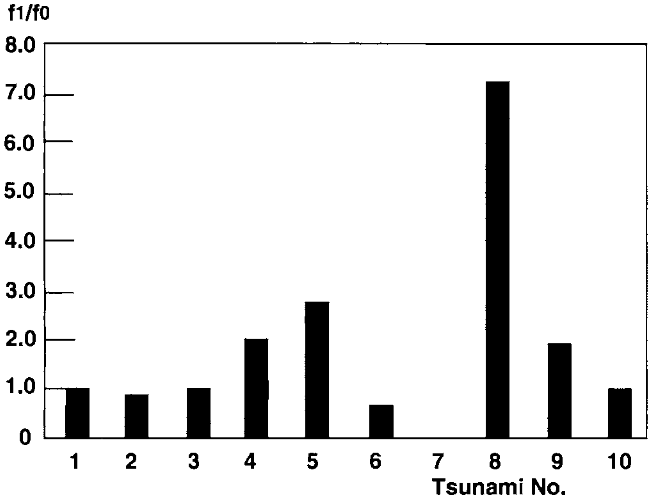

Figure 6 Ratio of peak beating frequency f_1 to dominant frequency of background f_0 versus each tsunami. Former and latter ones correspond to the open circle and open rectangle in the Figure 3, respectively.

ed in the background and this tsunami.

Discussion

As for the background oscillation at Chichijima island there is data obtained by Honda et al.(1908). They obtained the dominant period of 16-20 minutes (0.00104-0.00083 Hz) in the observation. We can use it under the assumption of no large topographic change around the tide station for about 100 years. Dominant frequencies of 0.0011 and 0.00094 Hz, which are obtained as two frequency components consisting of the beat, are within the range of 0.00083-0.00104 Hz. It is harmonious with our conclusion that one of frequencies consisting of the beat is equal to the fundamental or higher modes of the dominant frequency in the background spectrum.

Resolution of spectral peaks is decreased resulting from moving average of the spectral amplitude. The example is shown in the case of No.3 at Kuji tide station. Beating wave frequencies f1 and f2 are within one spectral peak as shown in Figure 3. These two frequencies are separated into two peaks of 0.0011 and 0.00102 Hz when we do not apply the moving average. Thus we recognize that the frequency difference of 0.00008 Hz is a limit to resolve the peak in this method.

One of the frequency components contributing to the beat is found in the highest limit of the band spectra and it is noticed that the spectra shows a sharp decay above the limit. This decay means a defect of frequency components higher than the limit in the incident tsunami. Therefore the limiting frequency is useful to estimate the highest frequency components included in the incident tsunami. A comparatively small value observed in No.8 at Tsuruga of 0.00074 Hz is possibly explained from a tsunami characteristic to a shallow sea of about 100m in depth (Aida et.al.,1964).

Conclusion

We obtained power spectra of beating tsunamis observed at tide stations in Japan and compared the beating frequencies with frequency components of the background spectra. It is found that one of the beating frequencies is equal to one of dominant modes in the background. Accordingly it is concluded that the beat is interpreted as a resonance of tsunami with the background oscillation,

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