ヨシカズエンド

| その他（別言語等）のタイトル | イオン型異方性のある1次元強磁性体
|-----------------------------|---------------------------------------------------|
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Two-dimensional ferromagnet with an anisotropy of single-ion character II

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Temperature renormalization of the single-ion anisotropy is studied in the limit of small anisotropy compared with the exchange constant. The renormalization factor is expressed in terms of the first moment \( <S^z> \), and then the excitation spectrum is approximated by the spin-wave one with the Ising-like anisotropy. This leads to reasonable results for 3D, but in 2D contains unphysical aspect in the profile of the magnetization curves.

In a previous paper (hereafter referred to as I),\(^1\) we have studied the effect of single-ion anisotropy of the form \(-D_0(S^z)^2\) \((D_0 > 0)\) in a two-dimensional ferromagnet. This anisotropy plays an important role in the system with competing interactions leading to the temperature-driven spin-reorientation transition, and have been studied extensively in low dimensional ferromagnets with dipolar interaction.\(^2-4\) In this brief report we give some comments on the results in I by adding attention to the temperature renormalization of the single-ion anisotropy.

We consider the same Hamiltonian as in I

\[
H = -\frac{1}{2} \sum_{lm} J_{lm} S_l \cdot S_m - D_0 \sum_i (S^z_i)^2. \tag{1}
\]

The equation of motion for \( S^+_i(t) \) under the RPA decoupling couples with \( \hat{S}^+_i(t) \equiv S^+_i(t)S^z_i(t) + S^z_i(t)S^+_i(t) \), and these two coupled equations of motion yield the excitation spectra

\[
\omega_k^\pm = \frac{1}{2}\{\mu(2c_0 - c_k) \pm \Gamma_k\} \tag{2}
\]
with \( \Gamma_k = \sqrt{\mu^2 c_k^2 + 4D_0(D_0 - Zc_k)} \) in units with \( J = 1 \), where \( c_k = 4\gamma_{k\parallel} + 2\delta\gamma_{k\perp} \), \( \gamma_{k\parallel} = (\cos k\parallel a + \cos k\perp a)/2, \gamma_{k\perp} = \cos k\parallel a, a \) the lattice constant. The lattice structure is assumed to be a square lattice (\( \delta = 0 \)) or a simple cubic lattice (\( \delta = 1 \)) as the representative of 2D or 3D ferromagnet.

To determine \( \mu = \langle S_i^z \rangle \) and \( Z = 3 < (S_i^z)^2 > -2 \), we use the Green’s function method developed by Devlin and equivalent one by other authors.\(^5\) We have

\[
< S_i^- S_i^+ > = \mu \Psi + Z \Phi \quad < S_i^- \tilde{S}_i^+ > = Z \Psi + \mu \Phi
\]  

(3)

where

\[
\Psi = \frac{1}{N} \sum_k \left\{ (1 - \frac{\mu c_k}{\Gamma_k})f(\omega_k^+) + (1 + \frac{\mu c_k}{\Gamma_k})f(\omega_k^-) \right\}
\]

(4)

\[
\Phi = 2D_0 \frac{1}{N} \sum_k \frac{1}{\Gamma_k} \left\{ f(\omega_k^+) - f(\omega_k^-) \right\}.
\]

(5)

Combining (3) with \( < S_i^- S_i^+ > = 4/3 - \mu - Z/3 \) and \( < S_i^- \tilde{S}_i^+ > = \mu - Z \), self-consistent equations to determine \( \mu \) and \( Z \) are obtained. We shall refer to this method as an exact one in what follows.

In the limit of small \( D_0 \) compared with the exchange constant \( J \), we may approximate \( \omega_k^\pm \) by

\[
\omega_1^\pm = \mu(c_0 - c_k + \alpha D_0) \quad \omega_2^\pm = \mu(c_0 - \alpha D_0)
\]

(6)

with \( \alpha = Z/\mu^2 \). Lower frequency parts of \( \omega_k^- \) and \( \omega_k^+ \) correspond respectively to \( \omega_1^- \) and \( \omega_2^+ \), and higher frequency parts of them correspond respectively to \( \omega_2^- \) and \( \omega_1^+ \), in which \( \omega_1^+ \) represents a spin-wave mode having Ising-like anisotropy of magnitude \( \alpha D_0 \). Then we have

\[
\Psi = 2\phi - \phi' \quad \Phi = \frac{\mu}{Z} \phi
\]

(7)

where

\[
\phi = \frac{1}{N} \sum_k f(\omega_1^+) \quad \phi' = 2\alpha D_0 \frac{1}{N} \sum_k \frac{f(\omega_1^-) - f(\omega_2^+)}{2\alpha D_0 - c_k}.
\]

(8)

The correlation functions \( < S_i^- S_i^+ > \) and \( < S_i^- \tilde{S}_i^+ > \) then become

\[
< S_i^- S_i^+ > = 2\mu \phi \quad < S_i^- \tilde{S}_i^+ > = 2Z \phi + \left( \frac{1}{\alpha} - Z \right) \phi'.
\]

(9)
Fig. 1 Temperature renormalization $g = \omega_{10}^{-1}/(\mu D_0)$ for $b: D_0 = 0.02, c: D_0 = 0.04$. Curve a represents $\alpha = \omega_{10}^{-1}/(\mu D_0)$.

At this stage we approximate $\alpha$ as obtained from the relation determined by setting $\phi' \equiv 0$. Then we get the renormalization factor of the single-ion anisotropy constant $D_0$: \cite{6,7}

$$\alpha = \frac{2 - \sqrt{4 - 3\mu^2}}{\mu^2}$$

(10)

一起 with the self-consistent equations

$$\mu = \frac{4(1 + 2\phi) + (1/\alpha - 4)\phi'}{4(1 + 3\phi + 3\phi^2) - 3(1 + 2\phi)\phi'}$$

(11)
\[ Z = \frac{4 - 3(1 + 2\phi)\phi' / \alpha}{4(1 + 3\phi + 3\phi^2) - 3(1 + 2\phi)\phi'} \quad (12) \]

which are equivalent to the equations (12) and (13) in I. Since equation (11) contains only \( \mu \) as an unknown, it is easy to solve in a self-consistent manner and moreover applicable to 2D system due to the existence of the anisotropy term in \( \omega_{1k} \) and \( \omega_{2k} \). The \( T_c \) and the value of \( Z \) at \( T_c \) by (11) and (12) are calculated as

\[ \frac{J}{k_B T_c} = \frac{6c_0}{4c_0 - D_0} g \approx \frac{3}{2} + \frac{3D_0}{8c_0} + \cdots \]

\[ Z = \frac{D_0}{c_0} \quad (13) \]

whereas those by the exact method are

\[ \frac{J}{k_B T_c} = \frac{3(4c_0 + 3D_0)}{4(2c_0 + D_0)} g \approx \frac{3}{2} + \frac{3D_0}{8c_0} + \cdots \]

\[ Z = \frac{4D_0}{4c_0 + 3D_0} \approx \frac{D_0}{c_0} + \cdots \quad (14) \]

where

\[ g = \frac{1}{N} \sum_k \frac{1}{c_0 - c_k + \frac{3}{4}D_0}. \]

On the other hand, the molecular-field approximation can be obtained by leaving out \( c_k \) in the exact method, and then

\[ < S_i^- S_i^+ > = 2\mu \phi_M + (Z - \mu)\phi'_M \quad < S_i^- S_i^+ > = 2Z\phi_M + (\mu - Z)\phi'_M \quad (16) \]

where

\[ \phi_M = f(\omega^+) \quad \phi'_M = f(\omega^+) - f(\omega^-) \quad (17) \]

where \( \omega^\pm = c_0 \mu \pm D_0 \). We see that equations (9) with \( \alpha \) defined by (10) coincide with equations (16) up to the first order in \( D_0 \).

In Fig.1 we plot curves \( g \equiv \omega_0^2 / (\mu D_0) \) versus \( \mu \) for \( D_0 = 0.02 \) and 0.04 in 2D case. The curve a represents \( \alpha = \omega_{10} / (\mu D_0) \) given by (10). For \( \mu \approx 0 \), the curves b and c deviate from the curve a. In this regime \( \omega_k^\pm \) is expressed as

\[ \omega_k^\pm \approx \pm \frac{1}{2} E_0 + \frac{1}{2} (2c_0 - c_k) \mu \pm \frac{c_k^2}{4E_0} \mu^2 + \cdots \quad (18) \]

with \( E_0 = 2\sqrt{D_0(D_0 - Zc_k)} \) and hence \( \omega_k^\pm \) cannot be a spin-wave mode near \( T_c \) in contrast to the mode \( \omega_{1k} \).

In the above we have seen that the single-ion anisotropy, if it is weak in comparison with \( J \), may be approximated by an effective renormalized Ising-like
Fig. 2 (a) Magnetization curves for 2D for three types of $h(\mu)$ shown in (b).
Fig. 3 (a) Magnetization curves for 3D for three types of $h(\mu)$ shown in (b).
one such as \( \omega_{1k} \), and that the temperature renormalization may be expressed in terms of the first moment \( \mu \). The most simple approximation would be to neglect the terms containing \( \phi' \) in (11) and (12), which corresponds to the decoupling of the single-ion Green’s function \( \ll S_i^+ S_i^+ + S_i^+ S_i^- (t); S_m^- \rr \ll \mu \alpha \ll S_i^- (t); S_m^- \rr \). Only the spin-wave mode \( \omega_{1k} \) appears and the method becomes analogous to that for the isotropic case.\(^8\) Let us introduce a model with an excitation \( \omega_{1k} = \mu (c_0 - c_k + h(\mu)) \) having an appropriate form of \( h(\mu) \) and solve a self-consistent equation

\[
\mu = \frac{1}{2\Phi} \quad \Phi = \frac{1}{N} \sum_k \coth \left( \frac{\beta \omega_{1k}}{2} \right)
\]

for \( S = 1/2 \) obtained by the Green’s function method. This model can also simulate, by taking \( h(\mu) = 0 \) at a certain \( \mu = \mu_n \), a temperature-driven spin-reorientation caused by competing perpendicular anisotropy such as \(-D_0(S_i^2)\)^2 studied here and the in-plane one caused by, for example, the Ising-like one \(-\frac{1}{2} \epsilon J_{mn} S_i^z S_m^z\) or the dipolar interaction. The magnetization curves for 2D for typical \( h(\mu) \) given in Fig.2(b) are plotted in Fig.2(a). In evaluating \( \Phi \) in (19), we analytically performed the integration for small \( k \) values. The most significant term is written up to an appropriate cutoff \( q_{\text{cut}} \) as

\[
\Phi = \frac{2T}{\pi \mu} \ln \left( \frac{q_{\text{cut}}^2}{4h(\mu)} \right) + \ldots.
\]

Since the temperature dependence of \( h(\mu) \) is expressed in terms of only the first moment \( \mu \), the logarithmic divergence occurring when \( h(\mu) \) tends to zero may be suppressed by \( T \), and hence the self-consistent equation (19) may have a solution approaching \( \mu_n \) defined by \( h(\mu_n) = 0 \) as \( T \) tends to zero. When \( h(\mu) = 0 \), the spin-wave spectrum in the neighborhood of \( k = 0 \) is proportional to \( k^2 \) and the system becomes isotropic Heisenberg one. This should lead to a loss of the magnetization due to two-dimensional fluctuations,\(^9\) but then the condition \( h(\mu) = 0 \) cannot be satisfied anymore. The corresponding magnetization curves in 3D case are shown in Fig.3. Even if \( h(\mu) = 0 \), small \( k \) integration in \( \Phi \) in (19) converges, leading to reasonable results in contrast to the 2D case. This implies that in 2D system with vanishingly small gap, the renormalization of the single-ion anisotropy cannot be approximated in the form of Ising-like one or by using only the first moment, and
we have to take an alternative approach beyond the simple renormalized method studied here in order to consistent with the low-dimensional fluctuations inherent in 2D or quasi-2D system.

References