The meson mass spectrum based on the relativistic two quark model

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相対論的二体模型による中間子の
質量準位

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要旨

クォーク模型によれば中間子は一個のクォークと一個の反クォークからできている。ここでは相対論的二体模型をこのクォーク模型に応用して中間子の質量準位を計算した。

従来のものに関しては実験によく合った結果が得られた。しかし近年続々と発見されている新しい重い中間子については S 状態と P 状態を同一のパラメータで記述するのは無理であった。

なお計算には当物理教室の電子計算機横河ヒューレットパッカード社のディスクトップコンピュータシステム45を使用した。鈴川教授はじめ当教室のスタッフの方々に感謝する。
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§ 1. Introduction

Since Gell-Mann and others proposed the quark model\textsuperscript{12} in which the three types of fundamental particles constitute the hadron, it is believed that a meson is constructed by one quark and one anti-quark. After that the fourth quark was theoretically predicted\textsuperscript{3}, and recently the fifth and the other quarks are introduced.

In the experiment the quark itself is never observed, but the new particles, the $J/\Psi$ meson series and $Y$ mesons, are successively discovered for a last half decade. The existence of the fourth quark which is named to be charm and the fifth quark named to be bottom gets reality by these new meson discoveries.

There are several attempts to calculate the meson mass spectrum by the two quark model with spin. The one takes some experimental meson masses as the input\textsuperscript{3}, and the other takes account of Coulomb plus linear potential and P-state splitting for the charmonium\textsuperscript{4}.

Recently Takaya\textsuperscript{5} proposed a relativistic model of the two particles with spin in the point form\textsuperscript{6}, in which only translation operators contain the interaction term of the Hook-type.

In this paper we calculate the mass spectrum of the meson which is the bound state of the quark and the anti-quark, based on the above relativistic two quark model. In the ordinary series, we get a good agreement with the experiment, in terms of a unique set of parameters without the input of experimental masses. In the new heavy meson series, however, we must calculate with different potential for the S-state and the P-state.

§ 2. Forms of relativistic dynamics

Relativistic dynamics are expressed in terms of ten dynamical variables $P_{\mu}$ and $M_{\mu\nu}$, where $P_{0}$ is the total energy of the system, $P_{i} (i=1, 2, 3)$ are the components of momentum and $M_{ij}$ are the components of total angular momentum. Dirac claass-
fied the description of dynamical systems into three types, according to physical conditions.

The first is called the instant form in which an instant in the four-dimensional relativistic picture is a flat three-dimensional surface. The second is the point form in which the three-dimensional surface is a hyperboloid. In the last front form the surface is light-cone.

From the relations between particular infinitesimal transformations of coordinate system, the ten quantities $P_{\mu}$, $M_{\mu\nu}$ satisfy the relations of the Lie algebra of Poincaré group, i.e.

\[
\begin{align*}
\{P_{\mu}, P_{\nu}\} &= 0 \\
\{M_{\mu\nu}, P_{\lambda}\} &= -g_{\mu\lambda}P_{\nu} + g_{\nu\lambda}P_{\mu} \\
\{M_{\mu\nu}, M_{\rho\sigma}\} &= -g_{\mu\rho}M_{\nu\sigma} + g_{\nu\rho}M_{\mu\sigma} - g_{\mu\sigma}M_{\nu\rho} + g_{\nu\sigma}M_{\mu\rho},
\end{align*}
\]

where $g_{\mu\nu} = 0$ if $\mu \neq \nu$ and $g_{00} = g_{11} = g_{22} = g_{33} = 1$.

For a dynamical system with several particles, the components of both $P_{\mu}$ and $M_{\mu\nu}$ contain the interaction terms in the instant form and the front form, but in the point form $P_{\mu}$ contains the interaction terms but $M_{\mu\nu}$ is simply the sum of the values for the individual particles.

§ 3. Relativistic two quark model

In this section we explain the relativistic two quark model, including spin, in the point form. We define following quantities

\[
\begin{align*}
G &= \frac{P_1 + P_2}{M}, \\
G_0 &= \frac{\omega_1 + \omega_2}{M} = \sqrt{G^2 + 1}, \\
K &= \frac{1}{2} (P_1 - P_2 - \frac{(P_1 - P_2, G)}{2G_0(1 + G_0)}) G - \frac{G}{G_0} \left(\frac{m_1^2 - m_2^2}{2M}\right), \\
q_1 &= \sqrt{m_1^2 + K^2}, \\
q_2 &= \sqrt{m_2^2 + K^2},
\end{align*}
\]

where $P_i$, $m_i$ and $\omega_i$ are momentum, mass and energy operator of the particle $i$ ($i=1,2$) respectively, and $M$ is the total mass in the center of mass system, $G$ denotes velocity of the center of mass, $K$ is interpreted as the relative momentum of the two particles in the center of mass system and $q_i$ is the energy of the particle $i$.

In terms of these quantities, the generators of the Poincaré group without inter-
actions are written
\[ P_i = M G_i, \]
\[ J = -i G \times \nabla + L + S_1 + S_2, \]
\[ N = i G \cdot \nabla + G \times L + N(\text{spin}), \]
\[ N(\text{spin}) = \frac{K \times S_1}{G_0 q_1 + (G \cdot K) + m_1} - \frac{K \times S_2}{G_0 q_2 - (G \cdot K) + m_2} + \frac{q_1 + (G \cdot K) / (G_0 + 1)}{G_0 q_1 + (G \cdot K) + m_1} \]
\[ (G \times S_1) + \frac{q_2 - (G \cdot K) / (G_0 + 1)}{G_0 q_2 - (G \cdot K) + m_2} (G \times S_2), \] (3)
where \( s_i \) is the intrinsic spin of the particle \( i \),
\[ G_0 = (G_0, G), \]
\[ L = -i K \times \nabla K. \]

\( L \) denotes the inner angular momentum operator. \( J \) corresponds to \((i, j)\) components of \( M_{\mu\nu} \) and represents the space rotation operator, and \( N \) corresponds to \((0, i)\) components of \( M_{\mu\nu} \) which is the space-time rotation.

Assuming that the interaction terms depend only on the relative coordinate of two particles, we put the translation operators \( P_i \) in the form
\[ P_i = G_i m, \]
where \( m \) is the mass operator containing the interaction terms. But the other rotational operators are the same as those of Eq. (3) in the point form.

In order that the new operators satisfy the commutation relations of the Poincaré group, \( m \) must obey the following relations,
\[ [G_i, m] = 0, \]
\[ [J_i, m] = 0, \]
\[ [N_i, m] = 0. \]

(4)

For the sake of simplicity, we assume that two particles have the equal mass. Assuming that the interaction between these two particles is of Hook-type, we have the following mass operator which was originally obtained by Takaya
\[ m^2 = \epsilon_0 + 4k^2 + 4m^2 + \frac{g}{2} \left\{ -\frac{\partial}{\partial k^2} - \frac{2}{k} \frac{\partial}{\partial k} + \frac{L^2}{k^2} \right\} \]
\[ + 2 \left[ \frac{1}{\sqrt{k^2 + m^2} \ (\sqrt{k^2 + m^2 + m})} - \chi(k^2) \right] S \cdot L \]
\[ + \left[ \frac{1}{\sqrt{k^2 + m^2} \ (\sqrt{k^2 + m^2 + m})} - \chi(k^2)^2 \ (k^2 S^2 - (k \cdot S)^2) \right], \]

(5)

where \( g \) is the coupling constant, \( \chi(k^2) \) is an arbitrary function of \( K^2 = k^2 \), \( S = S_1 + S_2 \), and \( \epsilon_0 \) expresses the depth of Hook-potential at \( k = 0 \).
§ 4. Calculation and discussion

We calculate the meson mass using the operator in Eq. (5). We treat the spin-independent terms of the mass operator to be the unperturbed part and its eigenvalue is given by

\[ m_0^2 = \varepsilon_0 + 4m^2 + \sqrt{8g(2n + L + 3/2)} \]  

(6)

where \( n \) is a radial quantum number and \( L \) is an orbital angular momentum. Its eigenfunction is

\[ \psi_{nL}(x) = \sqrt{\frac{2n!}{\Gamma(n+L+3/2)}} \frac{k^{L+3/2}}{x^{L+1/2}} e^{-x^2/4} L_n^{(L+1/2)}(\kappa x^2) \]

where \( x = mk \), \( \kappa^2 = \frac{8m^4}{g} \), \( L_n^{(L+1/2)}(\kappa x^2) \) is the associated Laguerre's polynomial, and \( \Gamma(n+L+3/2) \) is the gamma function. Adding spin-dependent parts as the perturbations, we get an expectation value

\[ \langle m^2 \rangle = \varepsilon_0 + 4m^2 + \sqrt{8g} (2n + L + 3/2) \]

\[ + g \left[ \frac{1}{\sqrt{k^2 + m^2}} \left( \frac{1}{\sqrt{k^2 + m^2 + m}} \right) - \chi(k^2) S \cdot L \right] \]

\[ + \frac{g}{2} \left[ \frac{1}{\sqrt{k^2 + m^2}} \left( \frac{1}{\sqrt{k^2 + m^2 + m}} \right) - \chi(k^2) \right]^2 \left( k^2 S^2 - (k \cdot S)^2 \right) \]  

(7)

We calculate the spin-dependent effects by the first order perturbation, because the contribution of the higher order perturbed parts is small. Numerically integrating the right hand side of the above equation, we get the result in Table 1 for the ordinary meson series.

Here we have confined our calculation to the iso-vector mesons, but exclude \( \pi \)-meson, regarding it as the Goldstone boson. We put the potential \( \varepsilon_0 \) to be \(-1.45 \) GeV, u-quark and d-quark masses to be \( 0.37 \) GeV, and coupling constant \( g \) to be about \( 1/8 \). The last value is reasonable compared with the one expected from the Regge slopes. We have taken the arbitrary function \( \chi(k^2) \) to be zero. Because \( \rho \)-meson is measured most precisely among the mesons of this Table and is well described by the quark model, we take this value to fix the potential \( \varepsilon_0 \).

For \( \delta \)-meson and \( A_1 \)-meson, the differences between the calculated mass and the experimental one are rather large. But the \( \delta \)-meson may be more complicated quark-antiquark state, including K\( \bar{K} \) channel, and the masses of \( A_1 \)-meson, \( \rho' \)-meson and \( A_3 \)-meson are not certainly established. The other meson masses are in good agreement with experiments. We take \( \rho' \)-meson to be in the first excited S-state, and the order of \( A_3 \)-meson and \( \rho' \)-meson is changed in our calculation.

For the new meson (c\( \bar{c} \)) series, a good agreement is obtained only when S-state and P-state are calculated with different potential \( \varepsilon_0 \). A more detaild discussion for the (c\( \bar{c} \)) system will be given in the next paper.
Table 1. The calculated meson masses of the ordinary series

<table>
<thead>
<tr>
<th>Meson</th>
<th>Theoretical</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S^{*}L_{J}$</td>
<td>mass square (GeV)$^2$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$^3S_1$</td>
<td>0.600</td>
</tr>
<tr>
<td>$\delta$</td>
<td>$^3P_0$</td>
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<tr>
<td>$A_1$</td>
<td>$^3P_1$</td>
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<tr>
<td>$B$</td>
<td>$^1P_1$</td>
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<tr>
<td>$A_2$</td>
<td>$^3P_2$</td>
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</tr>
<tr>
<td>$\rho'$</td>
<td>$^2S_1$</td>
<td>2.571</td>
</tr>
<tr>
<td>$A_3$</td>
<td>$^1D_2$</td>
<td>2.514</td>
</tr>
<tr>
<td>$g$</td>
<td>$^3D_3$</td>
<td>2.903</td>
</tr>
</tbody>
</table>

The choice of parameters in Eq. (7) are as follows: $\epsilon_0 = -1.45$GeV, $m = 0.37$GeV, $g = 0.122$.

Acknowledgement

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Reference